Evaluation of Hedge Effectiveness Tests

Angelika C. Hailer, Siegfried M. Rump*

Abstract. According to IAS 39 or FAS 133 an a posteriori test for hedge effectiveness has to be implemented when using hedge accounting. Both standards do not regulate which numerical method has to be used.

A number of hedge effectiveness tests have been published recently. Such tests are of different quality, for example not all of them can deal with the problem of small numbers. This means a test might determine an effective hedge to be ineffective, a scenario which would increase the volatility in earnings. Therefore, it seems useful to have criteria at hand to discriminate and assess hedge effectiveness tests.

In this paper, we introduce such objective criteria, which we develop according to our understanding of minimum economic requirements. They are applicable to tests based on market values of two points in time as well as on tests based on time series of market values.

According to our criteria we compare common tests like the dollar offset ratio, regression analysis or volatility reduction, showing strengths and weaknesses. Finally we develop a new adjusted Hedge Interval test based on our previous one (2003). Our test does not show weaknesses of other effectiveness test.

Keywords: Hedge Accounting, Assessment of Effectiveness Tests, FAS 133, IAS 39

Abbreviations: FAS – Financial Accounting Standard, IAS – International Accounting Standard, GG – hedged item (Grundgeschäft), SG – hedging instrument (Sicherungsgeschäft)

1 Introduction

The increasing importance of derivatives and hedges in today's economy presents a number of challenges for accounting. Significantly, these challenges have not led to definitive guidelines directing accounting activities but merely to a regulatory framework defined in FAS 133 for US-GAAP (United States Generally Accepted Accounting Principles) and in IAS 39 for the International Accounting Standards.

Both require derivatives to be reported at fair value on the balance sheet. However, to avoid an increase in the volatility of earnings due to changes in their market values, they also allow for derivatives to be recognized in the reporting as part of a hedge.

Where the latter option is being chosen, a number of conditions have to be met. One of these conditions is an a posteriori test for hedge effectiveness. A

*Institute for Computer Science III, Hamburg University of Science and Technology, hailer@tu-harburg.de, rump@tu-harburg.de

Table 1: Definitions provided in IAS 39.

A **hedged item** is an asset, liability, firm commitment, highly probable forecast transaction or net investment in a foreign operation that (a) exposes the entity to risk of changes in fair value or future cash flows and (b) is designated as being hedged.
A **hedging instrument** is a designated derivative or (for a hedge of the risk of changes in foreign currency exchange rates only) a designated non-derivate financial asset or non-derivative financial liability whose fair value or cash flows are expected to offset changes in the fair value or cash flows of a designated hedge item.
Hedge effectiveness is the degree to which changes in fair value or cash flows of the hedged item that are attributable to a hedged risk are offset by changes in the fair value or cash flows of the hedging instrument.

number of different tests are being used in practice; however, so far no criteria for assessing the quality of these tests have been established.

This paper starts by defining measurable criteria for the evaluation of effectiveness tests. These criteria are shown to be meaningful and most natural.

Existing tests can be broadly divided into those which are based on two points of time and those which are based on time series. We begin by examining a number of tests based on two points of time (dollar offset ratio, intuitive response to the small number problem, Lipp modulated dollar offset, Schleifer-Lipp modulated dollar offset, Gürtler effectiveness test, hedge interval). We then continue by looking at tests based on time series (expansion of test based on two dates, linear regression analysis, variability-reduction, and volatility reduction measure).

As we will demonstrate, the existing tests fail to meet the criteria developed in Section 3. However, by modifying the Hedge Interval Test discussed among others in Section 4, it is possible to obtain a test for hedge effectiveness which fulfills all of those criteria, as will be shown in Section 5.

2 Hedge Effectiveness according to IAS 39 and FAS 133

According to IAS we use the definitions of Table 1, i.e. in short a hedged item is the asset or liability responsible for the risk and a hedging instrument is the derivate to offset that risk. Additionally we consider a hedge position to be the added market value of hedged item and hedging instrument.

A hedging relationship qualifies for hedge accounting if and only if certain conditions defined in IAS 39 §88, or in FAS 133 §20, 21 resp. §28, 29 are met. One central condition part of both standards is the a posteriori assessment of hedge effectiveness as determined in IAS 39, §88 (e): "The hedge is assessed on an ongoing basis and determined actually to have been highly effective throughout the financial reporting periods for which the hedge was designated."

Equivalently, retrospective evaluations are summarized in the Statement 133 Implementation Issue No. E7 as follows:

"At least quarterly, the hedging entity must determine whether the hedging relationship **has been** highly effective in having achieved offsetting changes in fair value or cash flows through the date of the periodic assessment."

For considerations on the determination of market values and the use of marked-to-market values or clean values we refer to Coughlan, Kolb and Emery (2003). We concentrate on the problem of choosing an appropriate effectiveness test for a hedge position for which the fair values are already determined.

The dollar offset ratio is a common and probably the most simple method for asserting hedge effectiveness. It is explicitly explained in IAS 39 AG105. According to this measurement a hedge is regarded as effective if the quotient of changes of hedge item and hedging instrument is in the interval $[\frac{4}{5}, \frac{5}{4}]$.

Both standards explicitly state that other methods can be used as well: As part of FAS 133 §62 it is specified that the "appropriateness of a given method of assessing hedge effectiveness can depend on the nature of the risk being hedged and the type of hedging instrument used." The equivalent formulation in IAS 39 AG107 is that this "Standard does not specify a single method for assessing hedge effectiveness. The method an entity adopts for assessing hedge effectiveness depends on its risk management strategy." Ultimately the decision of which method is reasonable is left to the corporation's auditors.

As no details for these methods are provided by the standards, a number of different tests have been published recently. To compare these hedge effectiveness tests, we develop assessment criteria in the following section.

3 Criteria for Hedge Effectiveness Tests

The purpose of the effectiveness test is to check whether the market development of hedged item and hedging instrument are almost "fully" offsetting each other. As stated in FAS 133 §62 this "Statement requires that an entity define at the time it designates a hedging relationship the method it will use to assess the hedge's effectiveness in achieving offsetting changes in fair value or offsetting cash flow attributable to the risk being hedged."

Even more explicitly this is addressed as part of FAS 133 §230: "A primary purpose of hedge accounting is to link items or transactions whose changes in fair values or cash flows are expected to offset each other. The Board therefore decided that one of the criteria for qualification for hedge accounting should focus on the extent to which offsetting changes in fair values or cash flows on the derivative and the hedged item or transaction during the term of the hedge are expected and ultimately achieved."

So our first criterion should measure the degree of *offsetting.* This also implies that if concurrent market values in hedged item and hedged instrument are observed, a hedge should be regarded ineffective. As defined in both standards this means the relative deviation of the differences of changes of fair values to a perfect hedge should be limited.

Gürtler (2004) explains that the maximum possible gain or loss in the value of the hedge position should be limited. For example, a hedge where the difference in the market value of the hedging instrument is always -80% of the difference in the hedged item, is regarded as effective under the dollar offset ratio. But for a large loss in the market value of the hedged item the reduction in the value of the hedge position would be significant. In other words this "problem of *large numbers"* has to be avoided, the second criterion.

The implementation of the dollar offset ratio frequently generates the difficulty known as the "problem of *small numbers".* When there are just small changes in the market value of the hedged item, i.e. when the denominator gets small, the dollar offset ratio often indicates ineffectiveness although the tested hedge could be perfect. Therefore one third criterion should focus on this case: Equality of increase and decrease must not be strict, when nearly no changes in the market value of hedged item and hedging instrument are observed. In this case the hedge should be measured as effective.

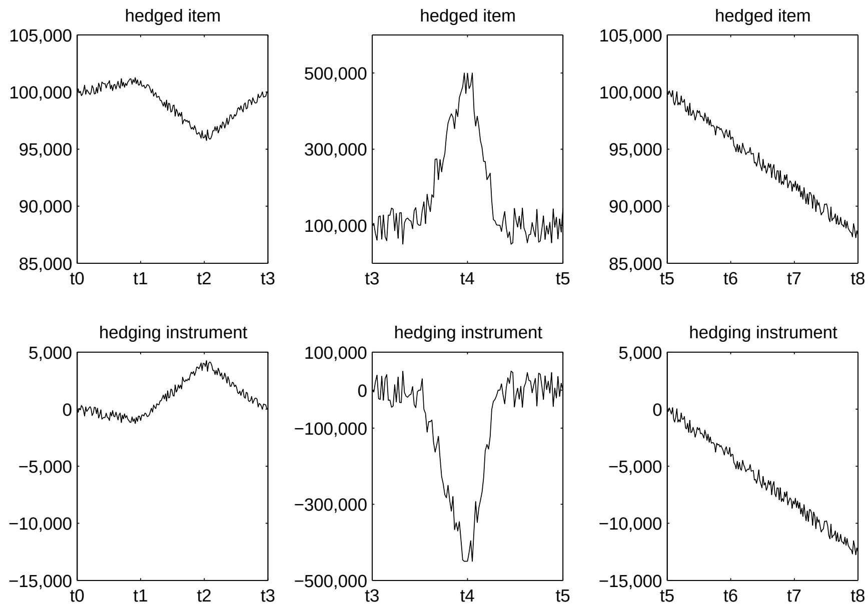
Offsetting does mean that if one of the market values of hedged item or hedging instrument decreases the other will increase, and vice versa. This implies *symmetry* with respect to hedged item and hedging instrument, i.e. if for a hedge position the value of the hedged item increases about e.g. 100,000 US$ and the loss in the hedging instrument is 120,000 US$, then the same result should be obtained of the effectiveness test as it would be obtained for a gain of 100,000 US$ in the market value of the hedging instrument and a decrease of 120,000 US$ for the hedged item.

In addition, significantly over- or under-hedged positions should not be regarded as effective, which means no bias with respect to gain or loss in the hedge position should occur. The effectiveness test should have the same result when applied to differences in market values of both hedged item and hedging instrument as when applied to their negative differences, i.e. in the above example a decrease in the value of the hedged item about e.g. 100,000 US$ and an increase of 120,000 US$ in the value of the hedging instrument should lead to the same result.

Furthermore, a test should be expected to be *scalable,* so that if a hedge relationship is effective, then using the same percentage of both hedged item and hedging instrument should result in an effective hedge as well. Analogously this should hold for an ineffective hedge. Consequently, the amount of the hedging position should not influence the result of the test. If the test is not scalable, separating a hedge position in two parts could lead to one effective and one ineffective part, which would not be reasonable.

Figure 1 on page 5 contains an adjustment of our example (Hailer and Rump, 2003), which was expanded by Gürtler (2004). It contains three main stages. From t_0 to t_3 the hedge is obviously effective, from t_3 to t_5 it illustrates the problem of extreme losses as mentioned by Gürtler (2004) and from t_5 on the development of the market values is concurrent, i.e. we have an increase instead of an offsetting of the risk, which means the hedge is strongly ineffective.

We investigate a number of tests in the following sections and summarize the results for this example in Tables 2 and 4. Deviations from the expected results



	GG in US$	SG in US$
t_0	100.000,00	0,00
t_1	100.999,90	- 1.000,07
t_2	96.000,00	4.000,00
t_3	99.999,90	0.07
t_4	500.000,00	-450.000.00
t_5	100.000,00	0.00
t_6	95.833,00	-4,166.00
t_7	91.666,00	-8,333.00
t_8	87.500,00	-12.500,00

Figure 1: Illustration of a sample market value development of hedged item and hedging instrument which belongs to a perfect hedge for balance sheet dates t_0 to t_3. From t_3 to t_5 an probably rather theoretical extreme movement in the market value can be observed and from t_5 the market values are concurrent which obviously implies the hedge to be ineffective.

are emphasized. A cursive *no* implies to regard an effective hedge as ineffective and therefore cause the hedge position to be dissolved. So the volatility in earnings would be increased. Even worse it a cursive *yes,* which allows an ineffective hedge to qualify for hedge accounting. In this case, earnings or losses can be hidden in the hedge position.

For developing comparable criteria according to the objectives described above, we distinguish between effectiveness tests based on the market value on two points of time and tests based on time series of market values.

3.1 Tests Based on Two Points of Time

Let GG_t denote the market value of the hedged item at date t and let ΔGG denote the market value difference in the hedged item, let SG_t and ΔSG be defined analogously for the hedging instrument and GP_t and ΔGP analogously for the hedge position, i.e. $GP_t = GG_t + SG_t$.

As mentioned in Statement 133 Implementation Issue No. E8 (2000) the difference ΔGG and respectively ΔSG can be calculated at date t on a period-by-period approach as $\Delta GG = GG_t - GG_{t-1}$, or cumulatively as $\Delta GG = GG_t - GG_0$:

> "In periodically (that is, at least quarterly) assessing retrospectively the effectiveness of a fair value hedge (or a cash flow hedge) in having achieved offsetting changes in fair values (or cash flows) under a dollar-offset approach, Statement 133 permits an entity to use either a period-by-period approach or a cumulative approach on individual fair value hedges (or cash flow hedges)."

This citation "relates to an entity's periodic retrospective assessment and determining whether a hedging relationship continues to qualify for hedge accounting". As already stated we consider the a posteriori test of hedge effectiveness and do not refer to the measurement of actual ineffectiveness that has to be reflected in earnings according to FAS 133 §22 or §30. For these, the Standard requires calculations on a cumulative basis.

In general it is not advisable to use only local information for a global measurement. In this case, when relying on period-by-period information, small, slow changes of the market value of the hedge position, which are not recognized as significant, may sum over time. In accordance with Coughlan, Kolb and Emery (2003) and Finnerty and Grand (2002) the following considerations for measurements relying on data of two points of time are based on the use of cumulative differences.

In addition to the general consideration for effectiveness tests, we expect a hedge effectiveness test based on two points of time to be continuous in the sense that there are no unnatural limits for the transition from effectiveness to ineffectiveness. For example, if for a decrease in the hedged item of 100.01 US$ a hedge is regarded as effective for an increase in the market value of the hedging instrument of 80.01 US$ and as not effective for and increase of 80.00 US$, then we cannot understand a hedge with a decrease in the hedge item of 100.00 US$

to be effective for a range of changes in the market value of the hedging instrument from -100.00 US$ to 100.00 US$. We assume these marginal values to be unnatural, and therefore should be avoided. So we expect a *smooth transition* from effectiveness to ineffectiveness.

The degree of offsetting can be geometrically interpreted when plotting ΔSG against ΔGG as shown in Figure 2.

The objective of measuring offsetting is then expressed by the relation

$$\Delta SG \approx -\Delta GG \, .$$

Therefore a hedge is highly effective if the point with the coordinates ΔGG and ΔSG is on or near the northwest-southeast diagonal, the bisecting line of the second and fourth quadrant. To determine the degree of "nearness" is the necessary task of a hedge effectiveness test.

All known tests based on market values of just two dates can be illustrated in the plane spanned by the coordinates ΔGG and ΔSG as shown in Figure 2.

The area where a hedge is regarded effective can be defined by bounding functions

$$\underline{f} : I\!R \to I\!R \quad \text{and} \quad \overline{f} : I\!R \to I\!R \, .$$

They define a hedge to be effective if

$$\underline{f}(\Delta GG) \leq \Delta SG \leq \overline{f}(\Delta GG) \, .$$

The bounding functions of some of the known measurements contain constants that depend on the initial value of the hedge position $GP_0 = GG_0 + SG_0$. When we need to indicate this parameterization for the underlying hedge position GP_0 we write $\underline{f}_{GP_0}$ and $\overline{f}_{GP_0}$ as well. In the figures the effective area is marked in grey.

For example, using the dollar offset ratio a hedge is effective if the point with the coordinates ΔGG and ΔSG is part of two cones, see gray area in Figure 4

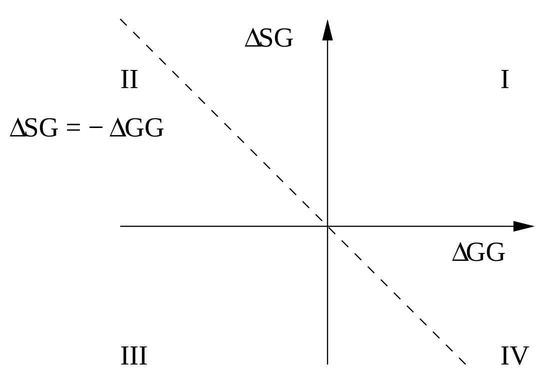


Figure 2: The plane spanned by ΔGG and ΔSG used for geometrical interpretation of effectiveness tests. The coordinates of a perfect hedge are on or near the dashed line.

on page 13. In this case the problem of small numbers is visible by the tolerance being very small close to the origin, the vertex of the cones.

From this illustration and the general considerations on the objectives of an effectiveness test at the beginning of this section, we deduce the following measurable criteria:

Criterion 1 *We assume an effectiveness test to comply with the following requirements:*

(i) **Offsetting:** *The surface indicating effectiveness should contain all effective hedges which are represented as part of the northwest-southeast diagonal. The relative deviation of this line should be limited for all $\Delta GG \in \mathbb{R}$.*

(ii) **Large numbers:** *The maximum gain or loss in the hedge position should be limited. So the absolute deviation of the northwest-southeast diagonal should be limited for all $\Delta GG \in \mathbb{R}$.*

(iii) **Small numbers:** *To avoid numerical problems the area should at no point have a vanishing "diameter", i.e. for arbitrary ΔGG*

$$|\overline{f}(\Delta GG) - \underline{f}(\Delta GG)| > \delta$$

for fixed $\delta \geq 0$.

(iv) **Symmetry:** *The surface should be symmetric to the northwest-southeast diagonal for symmetry in gain and loss of the hedge position and to the southwest-northeast diagonal to guarantee symmetry in hedged item and hedging instrument.*

This means, assumed the functions $\overline{f}$ and $\underline{f}$ are invertible the equations

$$\underline{f}(x) = \underline{f}^{-1}(x) \quad \text{and} \quad \overline{f}(-\underline{f}(x)) = -x$$

should hold true for all $x \in \mathbb{R}$.

(v) **Scalability:** *For all percentages $\alpha \in (0, 1]$ we should obtain*

$$\underline{f}_{\alpha\, GP_0}(\alpha \cdot \Delta GG) = \alpha \cdot \underline{f}_{GP_0}(\Delta GG)$$

and

$$\overline{f}_{\alpha\, GP_0}(\alpha \cdot \Delta GG) = \alpha \cdot \overline{f}_{GP_0}(\Delta GG)$$

for all $\Delta GG \in \mathbb{R}$. When regarding the functions $\underline{f}$ and $\overline{f}$ mathematically correct as functions of two parameters GP_0 and ΔGG, i.e. $\underline{f}, \overline{f} : \mathbb{R}^2 \to \mathbb{R}$, then $\underline{f}$ and $\overline{f}$ are said to be homogeneous of degree 1.

(vi) **Smooth transition:** *The transition between an effective and ineffective hedge should be natural, which implies the bounding functions $\underline{f}$ and $\overline{f}$ to be continuous.*

These criteria are independent of each other, which means it is not possible to deduce one from the other. So investigating an effectiveness test, all of the criteria (i) to (vi) have to be considered. In Section 4.1 we apply these to the main effectiveness tests known.

3.2 Tests Based on Time Series

According to IAS 39, AG106 effectiveness "is assessed, at a minimum, at the time an entity prepares its annual or interim financial statements." And more detailed as part of FAS 133, §20 (b) and §28 (b), an "assessment of effectiveness is required whenever financial statements or earnings are reported, and at least every three month." Therefore, we focus on assessments on a quarterly basis.

The first time a hedge position fails the hedge effectiveness test, it has to be dissolved as determined as part of IAS 39, AG113: "If an entity does not meet hedge effectiveness criteria, the entity discontinues hedge accounting from the last date on which compliance with hedge effectiveness was demonstrated." An equivalent explanation can be found in FAS.

Applying one of the methods based on two points of time quarterly indirectly includes historical data to the measurement. But this still incorporates only quarterly market values to the test. Thus, more detailed statistical approaches have been developed which are applicable to input data of time series of market values. The main idea is that the evaluation of the effectiveness of a hedge can be optimized when using as much information as available.

A problem appears in day to day business in the generation of these time series: IT-systems used for accounting purposes are often designed for punctual evaluations on reporting days. And even if accounting systems are able to deal with daily values, according to the securities used for the hedge daily market values may not be available.

Further on it seems to be common consent that statistical tests should be based at least on some 30 data points. In addition, for certain statistics the time intervals used should correspond to the hedged horizon as explained by Kawaller and Koch (2000):

> "Unfortunately, the need to use either quarterly price changes or price changes measured over the same time frame as the hedged horizon is common to any method of statistical analysis."

Using quarterly data this would imply historical data for at least seven years from inception of the hedge before a test could be applied, in contradiction to the assessment recommended on an ongoing basis by the standards. So the statistical requirements often cannot be fulfilled because of a lack of data.

Again the problem of small numbers may occur: For illustration we expand the example proposed by Kalotay and Abreo (2001): They consider a 100 US$ million bond hedged with an interest rate swap and a $10,000$ US$ rise in the value of the bond as well as a fall of $4,000$ US$ in the value of the swap. We assume the initial value of the hedging instrument to be zero at inception of the hedge. According to the dollar offset ratio this hedge is ineffective, which contradicts Kalotay and Abreo's statement that "the net change of US$ $6,000$ is a miniscule 0.006% of the face amount".

We construct a time series of 61 dates, where the market values of t_0 and t_{60} are those suggested by Kalotay and Abreo. The other dates are interpolated

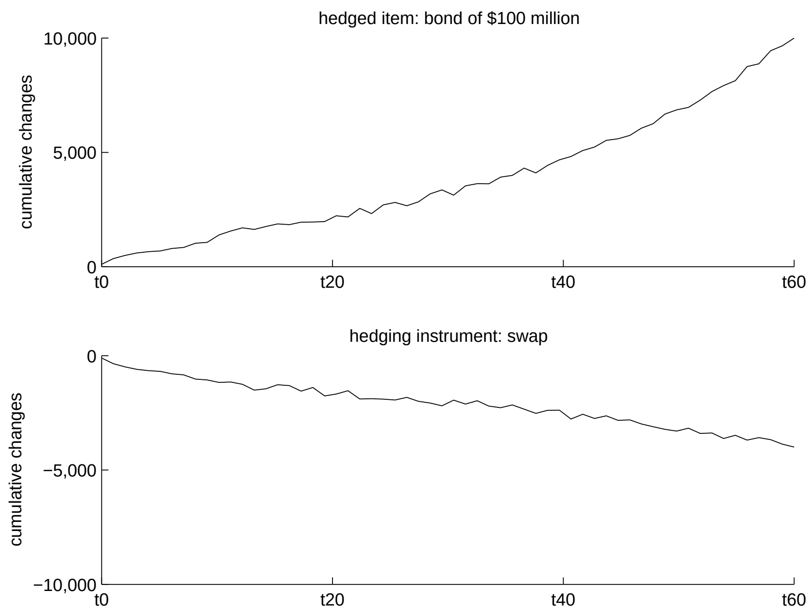


Figure 3: Illustration of the adjusted example of Kalotay and Abreo (2001) representing a market development for 60 days where nearly no changes can be observed. All tests evaluated in Section 4.2 result in an ineffective hedge. According to Kalotay and Abreo "the net change of $\$6,000$ is a miniscule 0.006% of the face amount", and therefore the hedge should be regarded as effective.

regarding a randomly perturbed logarithmic increase for the market value of the hedged item and decrease for the hedging instrument, as illustrated in Figure 3.

In day to day business this effect due to unchanged market values will occur less often when using daily market values for a larger time period, as they can be expected to have significant changes. We show in detail in the next section that a number of known measurements based on time series result in an ineffective hedge for this example. So the problem of small numbers is not necessarily avoided, although the probability of occurance is reduced compared to the dollar offset ratio.

Common to all known statistical measurements based on time series is the fact that the influence of the offsetting ratio on one point of time is reduced. Therefore, the management decision, whether or not to regard a hedge as effective even if it gets ineffective for single points in time should be discussed in advance.

Assume we have n dates where market values are available. For the dates $i = 1, \dots, n$ let ΔGG_i denote the cumulative difference in the market value of the hedged item i.e.

$$\Delta GG_i = GG_i - GG_0$$

or the period-by-period difference

$$\Delta GG_i = GG_i - GG_{i-1} ,$$

and for the hedging instrument $\Delta SG_i = SG_i - SG_0$ or $\Delta SG_i = SG_i - SG_{i-1}$, respectively.

Let $\overrightarrow{\Delta GG}$ denote the n-dimensional vector containing all ΔGG_i and $\overrightarrow{\Delta SG}$ the n-dimensional vector containing all ΔSG_i. For adjusting the criteria for two points to higher dimensions as necessary for time series, we introduce the effectiveness test function

$$T : I\!R^{2n} \to \{0,1\} : \; T(\overrightarrow{\Delta GG}, \overrightarrow{\Delta SG}) \to \{\text{not effective} \; , \; \text{effective}\} \; .$$

FAS 133 does not detail the requirements for statistical measurements, but the following waring is contained in Implementation Issue No. E7:

> "The application of a regression or other statistical analysis approach to assessing effectiveness is complex. Those methodologies require appropriate interpretation and understanding of the statistical inferences."

Independently of this point we suppose the general criteria we have described for an effectiveness test at the beginning of this section should be fulfilled, regardless of the complexity of the test.

So analog to the effectiveness tests based on two points we can formulate measurable criteria:

Criterion 2 *Suppose* $n \in I\!N$ *and* $T : I\!R^{2n} \to \{0,1\}$ *is an effectiveness test for hedge accounting. Then* T *should have the following properties:*

(i) **Offsetting:** *The scatter plot of all points* $(\Delta GG_i, \Delta SG_i)$ *should be close to the northwest-southeast diagonal, i.e. the relative deviation of this line should be limited for all points.*

(ii) **Large numbers:** *For all points* $(\Delta GG_i, \Delta SG_i)$ *the maximum distance to the northwest-southeast diagonal, i.e. the absolute deviation, should be limited.*

(iii) **Small numbers:** *The problem of small numbers should be avoided. Therefore, if*

$$\max_{i \in \{1,\ldots,n\}} \{\max\{|\Delta GG_i|, |\Delta SG_i|\}\} \leq c$$

for a constant c *which may depend on the initial value of the hedge position, i.e.* $c = c_{GP_0}$ *the hedge should be regarded as effective.*

(iv) **Symmetry:** *Using the points with the coordinates* $(\Delta SG_i, \Delta GG_i)$ *for testing effectiveness should imply the same result as the use of* $(\Delta GG_i, \Delta SG_i)$*, i.e.*

$$T(\overrightarrow{\Delta GG}, \overrightarrow{\Delta SG}) = T(\overrightarrow{\Delta SG}, \overrightarrow{\Delta GG}) \ ,$$

and symmetry with respect to gains and losses should be fulfilled, i.e.

$$T(\overrightarrow{\Delta GG}, \overrightarrow{\Delta SG}) = T(-\overrightarrow{\Delta GG}, -\overrightarrow{\Delta SG}) \ .$$

(v) **Scalability:** *Let* $\alpha \in (0, 1]$. *Then the property*

$$T(\overrightarrow{\Delta GG}, \overrightarrow{\Delta SG}) = T(\alpha \cdot \overrightarrow{\Delta GG}, \alpha \cdot \overrightarrow{\Delta SG})$$

should hold true.

In Section 4.2 we apply these criteria to the main effectiveness tests known, which are based on times series of market values. The results for all tests investigated are summarized in Table 5. In this case we expect a hedge at time t_5 to be not effective, as the decision is based on the time period from t_4 to t_5, which contains the problem of large numbers at the beginning.

According to standard statistical notation we use the following definitions: Let $\overline{X} = \sum_{i=1}^{n} x_i$ denote the mean value of $x_1, \ldots, x_n$.

For n observation dates let x_i and y_i denote market values or changes in market values. Then the empirical variance σ_x^2 and the empirical covariance σ_{xy}^2 are defined as

$$\sigma_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(x_i - \overline{X}\right)^2 \quad \text{and} \quad \sigma_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{X}) \cdot (y_i - \overline{Y}) \ ,$$

and the standard deviation can be estimated as $\sqrt{\sigma_x^2}$.

4 Evaluation of Common Hedge Effectiveness Tests

4.1 Tests Based on Two Points of Time

4.1.1 Dollar Offset Ratio

The dollar offset ratio is defined in the following effectiveness test.

Test 1 *A hedge is regarded effective if the quotient of changes of hedge item and hedging instrument is part of the interval* $[80\%, 125\%]$*, i.e. if*

$$-\frac{\Delta SG}{\Delta GG} \in [\frac{4}{5}, \frac{5}{4}] \ .$$

We summarize the results of this test for the example presented in Figure 1 in Table 2 on page 15. Geometrically a hedge is effective if the coordinates given by the values of ΔGG and ΔSG fall in the cones being spanned by the lines $\Delta SG = -\frac{4}{5}\,\Delta GG$ and $\Delta SG = -\frac{5}{4}\,\Delta GG$, the gray area in Figure 4.

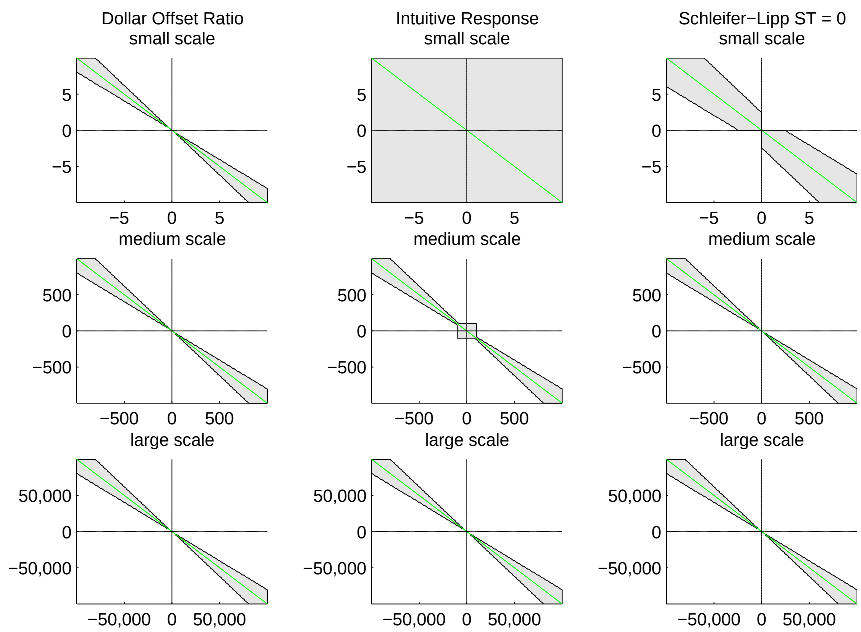


Figure 4: Comparison of the geometrical interpretation of the dollar offset ratio, the intuitive response and the Lipp Modulated dollar offset ratio. A hedge is effective if the coordinates of the changes of hedged item and hedging instrument are part of the grey area.

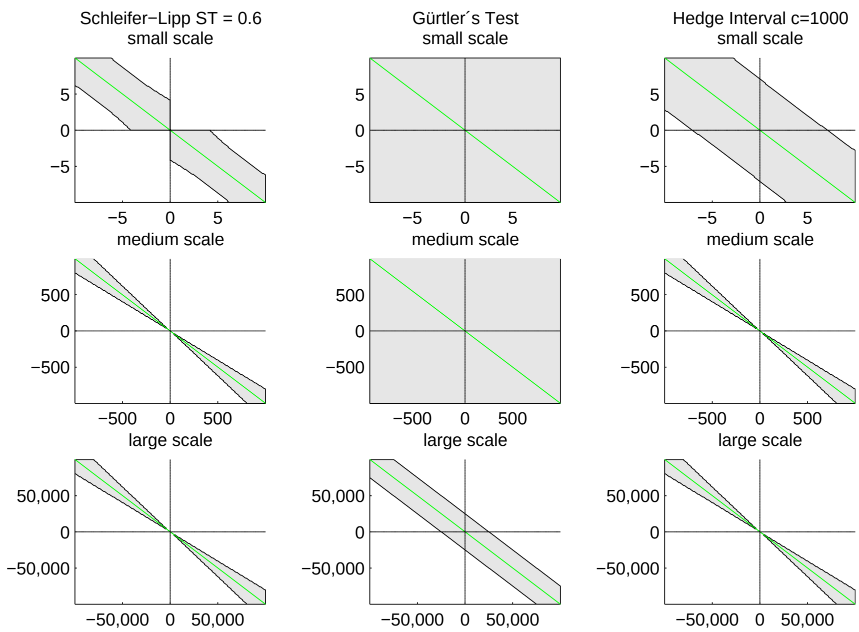


Figure 5: Comparison of the geometrical interpretation of the Schleifer-Lipp Modulated dollar offset ratio, the effectiveness test proposed by Gürtler and the hedge interval. A hedge is effective if the coordinates of the changes of hedged item and hedging instrument are part of the grey area. In particular regarding small or medium scale all points are effective when using the test of Gürtler.

Table 2: Results of the application of the hedge effectiveness tests based on two points of time for the hedge introduced in Figure 1 on page 5. Deviations from the expected results are emphasized.

	Expected to be effective	Dollar Offset Ratio $-\frac{\Delta SG}{\Delta GG}$	Dollar Offset Ratio effective	Intuitive Response $-\frac{\Delta SG}{\Delta GG}$	Intuitive Response effective	Lipp	Lipp effective
t_1	yes	100.02%	yes	100.02%	yes	100.02%	yes
t_2	yes	100.00%	yes	100.00%	yes	100.00%	yes
t_3	yes	70.00%	*no*		yes	99.70%	yes
t_4	no	112.50%	*yes*	112.50%	*yes*	112.50%	*yes*
t_5	yes	n/a	yes		yes	100.00%	yes
t_6	no	-99.98%	no	-99.98%	no		no
t_7	no	-99.99%	no	-99.99%	no		no
t_8	no	-100.00%	no	-100.00%	no		no

	Schleifer-Lipp $S_T = 0.6$	Schleifer-Lipp effective	Gürtler's Test $\frac{GP_T}{GP_0}$	Gürtler's Test effective	Hedge Interval based on $[\frac{4}{5}, \frac{5}{4}]$ x	Hedge Interval based on $[\frac{4}{5}, \frac{5}{4}]$ $\lvert x\rvert \leq 9$	Hedge Interval based on $[\frac{9}{10}, \frac{10}{9}]$ x	Hedge Interval based on $[\frac{9}{10}, \frac{10}{9}]$ $\lvert x\rvert \leq 19$
t_1	100.02%	yes	100.00%	yes	0.99	yes	0.97	yes
t_2	100.00%	yes	100.00%	yes	-1.00	yes	-1.00	yes
t_3	99.98%	yes	100.00%	yes	-0.04	yes	-0.17	yes
t_4	112.50%	*yes*	50.00%	no	-4.00	*yes*	-21.50	no
t_5	100.00%	yes	100.00%	yes	0.00	yes	0.00	yes
t_6		no	91.67%	*yes*	-80.99	no	-360.95	no
t_7		no	83.33%	*yes*	-80.99	no	-360.98	no
t_8		no	75.00%	*yes*	-81.00	no	-361.00	no

All criteria proposed in Section 3 except (ii), the maximum deviation for large values of ΔGG, and except (iii) are fulfilled:

As one can easily see the surface is symmetric, and the functions $\underline{f}$ and $\overline{f}$ are continuous. This is also true in the origin, even though at this point they are not differentiable. Scalability is fulfilled as any percentage would be canceled in the fraction.

As already mentioned as "problem of small numbers" for arbitrary $\delta > 0$ and $\Delta GG = 0$ we obtain

$$|\overline{f}(\Delta GG) - \underline{f}(\Delta GG)| = 0 < \delta\,,$$

so criterion (iii) is not fulfilled. And as explained by Gürtler (2004) the maximum loss of the hedge position is not limited as the distance of the bounding lines of the cones is arbitrarily large for large absolute values of ΔGG and ΔSG.

Out of these reasons modifications of the dollar offset ratio have been developed, which are investigated in the following sections.

4.1.2 Intuitive Response to the Small Number Problem

In the transition from accounting corresponding to the German accounting standards according to HGB (Handelsgesetzbuch) to IAS or US-GAAP, German companies encountered the problem of small numbers when implementing the dollar offset ratio.

One intuitive way to respond to this problem is to implement a fixed maximum value for changes in the market development of hedged item and hedging instrument, up to that a hedge is considered effective without further test. As far as we know this was proposed and accepted by one of the leading auditing firms.

Test 2 *A hedge is effective*

$$\begin{cases} \textit{without test} & \textit{for } \max\{|\Delta SG|, |\Delta GG|\} \le c \\ \textit{if } -\frac{\Delta SG}{\Delta GG} \in [\frac{4}{5}, \frac{5}{4}] & \textit{else.} \end{cases}$$

This test is only scalable, if the value of c is dependent on the value of the hedge position at inception of the hedge, in our example we take a value of 1‰, i.e.

$$c_{GP_0} = 0.001(GG_0 + SG_0) = 100\ .$$

As shown in Figure 4 on page 13 the functions $\underline{f}$ and $\overline{f}$ are not continuous, resulting in this example in an unexplainable transition of effectiveness for values of $\Delta GG = 100$ or $\Delta SG = 100$.

The problem of maximum deviation remains unsolved as already explained for the dollar offset ratio. But except criteria (ii) and (v), all others are fulfilled.

4.1.3 Lipp Modulated Dollar Offset

Further modification of the dollar offset are published by Schleifer (2001). These add basic values for the consideration of relative changes and discuss different values M_p for these. The Lipp modulated dollar offset is one measurement that is defined by the following test.

Test 3 *A hedge is regarded as effective if*

$$\operatorname{sgn}(\Delta SG) = -\operatorname{sgn}(\Delta GG) \quad \text{and} \quad \frac{|\Delta SG| + NT_A}{|\Delta GG| + NT_A} \in \left[\frac{4}{5}\,,\,\frac{5}{4}\right] ,$$

where NT_A is the absolute value of a noise threshold.

Schleifer (2001) proposes a definition of $NT_A = M_p \frac{NT_N}{10.000}$, where NT_N is a user-defined noise threshold and M_p depends on the hedged item's cash flows, in first-order approximation the present-value of one leg of the hedging instrument.

In the plane spanned by ΔGG and ΔSG we get for $\Delta GG \geq 0$ the inequalities

$$-\frac{NT_A}{4} - \frac{5}{4}\Delta GG \leq \Delta SG \leq \frac{NT_A}{5} - \frac{4}{5}\Delta GG \quad \text{and} \quad \Delta SG \leq 0 ,$$

and for $\Delta GG \leq 0$

$$-\frac{NT_A}{5} - \frac{4}{5}\Delta GG \leq \Delta SG \leq \frac{NT_A}{4} - \frac{5}{4}\Delta GG \quad \text{and} \quad \Delta SG \geq 0 .$$

In our calculation for Table 2 and Figure 5 we use a value of $NT_A = 10$.

One problem concerning the vertex of the cone remains unsolved: Nearly no changes in the market value of hedged item and hedging instrument could imply an ineffective hedge relationship. For example, a raise of both values of $\frac{1}{100}$ basis point can be interpreted as noise in the data.

Further on this test is only scalable if NT_A is a fixed percentage of GP_0. Obviously the bounding functions $\underline{f}$ and $\overline{f}$ are not continuous. So criteria (ii), (iii) and (v) are not fulfilled.

4.1.4 Schleifer-Lipp Modulated Dollar Offset

One suggested modification of this measurement is the Schleifer-Lipp modulated offset.

Test 4 *A hedge is effective, if*

$$\operatorname{sgn}(\Delta SG) = -\operatorname{sgn}(\Delta GG)$$

and for $S_T > -1$

$$\frac{|\Delta SG| \left(\frac{\sqrt{\Delta SG^2 + \Delta GG^2}}{NT_A}\right)^{S_T} + NT_A}{|\Delta GG| \left(\frac{\sqrt{\Delta SG^2 + \Delta GG^2}}{NT_A}\right)^{S_T} + NT_A} \quad \in \quad \left[\frac{4}{5}\,,\,\frac{5}{4}\right] .$$

For a parameter of $S_T = 0$ the Lipp modulated dollar offset is the same as the Schleifer-Lipp modulated dollar offset. Again we use a value of $NT_A = 10$ in our calculations.

This test is only scalable if NT_A is a fixed percentage of GP_0. It is similar to the Lipp modulated dollar offset and does not satisfy criteria (ii), (iii) and (v).

4.1.5 Gürtler Effectiveness Test

Gürtler (2004) develops a test from a risk-theoretical basis as he minimizes the maximum possible loss of the hedge position:

Test 5 *A hedge position is regarded as effective if and only if*

$$1 - \frac{\alpha}{a} \leq \frac{GP_t}{GP_0} \leq 1 + \frac{\alpha}{a} \ ,$$

where Gürtler suggests to use a value of $\frac{\alpha}{a} = 25\%$.

As Gürtler shows this is equivalent to a hedge being effective if

$$1 - \frac{\alpha}{a}\frac{GP_0}{|\Delta GG|} \leq -\frac{\Delta SG}{\Delta GG} \leq 1 + \frac{\alpha}{a}\frac{GP_0}{|\Delta GG|}$$

or equivalently

$$-\Delta GG - \frac{\alpha}{a}GP_0 \leq \Delta SG \leq -\Delta GG + \frac{\alpha}{a}GP_0 \ .$$

Geometrically these inequalities represent a fixed band around the northwest-southeast diagonal. The width of this band depends on the constant $\frac{\alpha}{a}$ and on the value of hedge position at inception of the hedge. For illustration see Figure 5 on page 14.

Up to a maximum loss of 25% of the hedge position GP_0, Gürtler's test always results in an effective hedge. Normally one would not observe such extreme movements in the market values as in our example in periods t_3 to t_5. Therefore, with this measurement a lot more hedges are qualifying for hedge accounting than when applying dollar offset ratio.

So this test satisfies all criteria but the first, which must not only be considered as the most important one according to the standards but also describes the main objective for implementing an effectiveness test.

This measurement is geometrically equivalent to the relative-difference test described by Finnerty and Grand (2002), according to which a hedge is effective if

$$\left| \frac{\Delta SG + \Delta GG}{GG_0} \right| \leq 3\% \ .$$

This method is investigated by Finnerty and Grand (2002) with a suggested bandwidth of $\sqrt{2} \cdot 0.03 \cdot GG_0$, whereas Gürtler proposes a bandwidth of $\sqrt{2} \cdot 0.25 \cdot (GG_0 + SG_0)$. In our example we obtain a bandwidth of $4,242.64$ US\$

in the first and of 35,355.34 US$ in the second case. With this narrower band the expected results would be obtained in our example, but the first criterion denoting the relative deviation is not met. This first criterium corresponds to the definition of offsetting in both Standards.

4.1.6 Hedge Interval

We presented a hedge interval with the following properties (2003):

- On a large scale the interval is essentially identical to the known dollar offset ratio.
- For small numbers the intersection of the cones of the dollar offset ratio is broadened.
- The transition from large to small is continuous.

The measurement is defined as follows:

Test 6 *A hedge is to be regarded as effective if and only if*

$$\left| \frac{40\ \Delta SG + 41\ \Delta GG}{\sqrt{\Delta GG^2 + c}} \right| \leq 9\ . \qquad (*)$$

The lower and upper bounding functions $\underline{f}$ and $\overline{f}$ are approximating the lines

$$\Delta SG = -\frac{5}{4}\Delta GG \qquad \text{and} \qquad \Delta SG = -\frac{4}{5}\Delta GG$$

for larger values and broadening the intersection of the two cones. The parameter c in $(*)$ determines the distance of the approximating function to the cones in the origin.

The test is not sensitive to changes in the parameter c, so c may vary within about an order of magnitude without causing too much change of behavior. If all balance sheet items regarded for hedging are about the same order of size the parameter c could be determined as a constant. According to the suggestions of Gürtler and to guarantee scalability we advice to introduce a dependency of c on the squared of the initial hedge position. For example, a value of $c = 10^{-7} \cdot GP_0^2$ seems appropriate in all real cases. For our calculations of the example described in Figure 1 we used this value of $c = 10^{-7} \cdot GP_0^2 = 1000$.

Interpreting the hedge interval from the view of numerical mathematics we look at relative error for large Δ and at absolute error for small Δ. The transition is smooth.

So all criteria except (ii), the maximum deviation for large values of ΔGG, are satisfied. As Gürtler stated it is common practice to use an interval for the dollar offset ratio of $[\frac{9}{10}, \frac{10}{9}]$ instead of $[\frac{4}{5}, \frac{5}{4}]$. At the end of this paper we present a generalization of this hedge interval to arbitrary underlying dollar offset intervals and adjust it to fulfill criterion (ii) as well.

4.2 Tests Based on Time Series

4.2.1 Expansion of Tests based on Two Dates

First of all a simple test which is easy to implement can be deduced from all of the measurements presented for two points of time as follows: A hedge is regarded as highly effective if and only if the coordinates of the differences ΔGG_i and ΔSG_i are part of the effective area for all dates i.

This is probably the strictest of the effectiveness test based on time series as it does not allow the hedge to get ineffective at one single point in the sense defined in the underlying two point criterion.

One modification of this measurement is presented by Coughlan, Kolb and Emery (2003), answering the question whether or not a hedge has to be effective on all dates where market values are available. They introduce a compliance level as

$$\text{Compliance level } = \frac{\text{Number of compliant results}}{\text{Number of data points}} ,$$

and suggest a threshold of 80%, i.e. to regard a hedge effective if the compliance level has a value greater than 80%.

These tests fulfill all of the corresponding criteria to their underlying method for two points of time, except the first two concerning offsetting: When a compliance level lower than 100% is used, the maximum relative and absolute deviation is not limited all of the time.

4.2.2 Linear Regression Analysis

In risk management calculations focusing on hedging strategies the hedge effectiveness can be tested using a linear regression on the ratio of the differences (Hull, 2003), even if Kalotay and Abreo (2001) refer to it as "arcane statistics such as R-squared".

This method is explicitly mentioned in IAS 39 F.4.4, where its application is detailed as follows: "If regression analysis is used, the entity's documented policies for assessing effectiveness must specify how the results of the regression will be assessed."

The linear regression is based on the equations

$$y_i = \widehat{\beta}_0 + \widehat{\beta}_1 \, x_i + e_i \ .$$

We refer to the version of Coughlan, Kolb and Emery (2003), where the independent variable x refer to the hedged item and the dependent variable y to the hedging instrument. This does not correspond to Kawaller and Koch (2000), who interchange the variables x and y.

For obtaining offsetting in differences of market value developments, i.e. $x_i = \Delta GG_i$ and $y_i = \Delta SG_i$ the value of the slope $\widehat{\beta}_1$ should be close to -1 and of the intercept $\widehat{\beta}_0$ close to 0. Further on, generally the adjusted R-squared, R^2, is determined and it seems to be common consent that it has to have a value greater than 80% for a hedge to be effective.

As stated in Finnerty and Grand (2002) "there is a tendency to interpret the Regression Method only by its adjusted R^2, although ineffectiveness can also appear in both the slope and intercept." Naturally we assume that for hedge effectiveness certain requirements for $\widehat{\beta}_0$ and $\widehat{\beta}_1$ have to be fulfilled. Otherwise, with a value of $\widehat{\beta}_1 = 1$ a perfectly ineffective hedge would be regarded effective, or with $\widehat{\beta}_0 \neq 0$ over- or under-hedging could be accepted as effective.

Coughlan, Kolb and Emery (2003) mention that it is important when using a linear regression to validate the statistical significance with a t-test, and suggest to use for this t-test a confidence level of 95%.

The standard t-test for a linear regression tests the hypotheses H_0, that the parameters β_0 and β_1 are equal to zero, to determine if the influence of these is statistically significant. If a probability less than 5 % is obtained than H_0 can be rejected. Coughlan, Kolb and Emery (2003) provide a sample output of a statistical tool for regression analysis. According to the data this standard t-test seems to have been used.

In the case of an effective hedge we expect to obtain values for β_0 close to 0 and for β_1 close to -1. So probably we will obtain the result that we can reject $H_0 : \beta_1 = 0$ and not reject $H_0 : \beta_0 = 0$. In our calculations for the example described in Figure 1 on page 5 we included this test for β_1 and were always able to reject $H_0 : \beta_1 \neq 0$.

We suppose in this case other hypotheses like $H_0 : \beta_1 \neq -1$ could be more appropriate. In addition, we would suggest to a priori set the intercept to zero in a regression based on changes of market values for a hedge position.

Nevertheless for the comparison of the tests we focus on the evaluation of $\widehat{\beta}_0$, $\widehat{\beta}_1$ and R-squared. Coughlan, Kolb and Emery (2003) use for the retrospective regression analysis an effectiveness threshold of -80% for the correlation and -0.80 to -1.25 for the slope. According to Kalotay and Abreo (2001) and Kawaller (2002) in our examples we regard a threshold of 80% for the R-squared.

The parameter for $\widehat{\beta}_0$ and $\widehat{\beta}_1$ are determined with standard statistic as

$$\widehat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \overline{X})(y_i - \overline{Y})}{\sum_{i=1}^{n}(x_i - \overline{X})^2} = \frac{\sigma_{xy}}{\sigma_x^2} \qquad \text{and} \qquad \widehat{\beta}_0 = \overline{Y} - \widehat{\beta}_1 \overline{X} \ .$$

The R-squared can be determined with the empirical variances and covariance as

$$R^2 = \frac{\sigma_{xy}^2}{\sigma_x^2 \sigma_y^2} \ .$$

For the linear regression different dependent and independent variables are discussed by Kawaller and Koch (2000) when investigating a priori hedge effectiveness tests. The main concern is whether regression should be applied to data on price levels or on price changes. No method is directly recommended by Kawaller and Koch (2000):

> "This discussion might suggest that the appropriate indicator of hedge effectiveness should be the correlation of price *levels*, as opposed to price *changes*, but this conclusion is similarly flawed. The

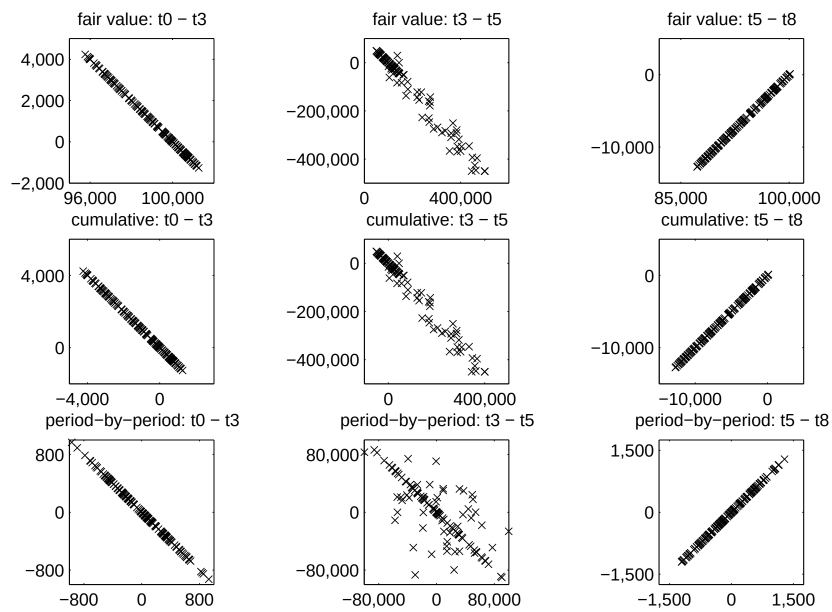


Figure 6: Scatter plots for the linear regression for the example introduced in Figure 1 for the different dependent and independent variables.

> statement that two price levels are highly correlated does not necessarily imply a reliable relationship between their price *changes* over a particular hedge horizon, which is the issue of concern for the FASB."

For price changes it has to be further distinguished between price changes on a period-by-period assessment or cumulative, i.e. calculating all differences to the inception of the hedge. We do not consider price changes based overlapping periods for the evaluation of the effectiveness tests based on time series. For a discussion we refer to Kawaller and Koch (2000).

In summary, we use the following simplified criteria, even if no explicit thresholds for β_0 are included in this test.

Test 7 *A hedge is regarded effective, if and only if a linear regression, which can be executed on fair values, cumulative or period-by-period changes results in a value*

$$\widehat{\beta}_1 \in [-\frac{4}{5}, \frac{5}{4}] \quad \textit{and} \quad R - squared \geq 80\,\% \,,$$

and if $\widehat{\beta}_0$ is sufficiently small for regression based on changes and close to the value of the initial hedge position for regressions based on fair values.

In Table 4, we compare the results of these three types of regression analysis, and in Figure 6 we illustrate the difference in the data for our example introduced in Figure 1 with scatter plots.

All these types of regression measure offsetting but do not meet the criteria of large numbers. A perfect linear dependency of the data with a value $R^2 = 100\%$, a slope $\beta_1 = -4/5$, and additionally an intercept of zero for market value changes is under the regression test an effective hedge, but the loss of the hedge position is not limited.

The problem of "small numbers" is not avoided as well: When there are nearly no changes in market value of hedged item and hedging instrument the coordinates for the points are all close to each other. So a line approximating these must not necessarily exist, even if the hedge is almost perfectly effective. For the adjusted example of Kalotay and Abreo (2001) we obtained ineffectiveness, the detailed results are summarized in Table 3.

The measurements based on regression analysis as defined in Test 7 are not symmetric but scalable. So only the first and last criterium are fulfilled.

Table 3: Results of the application of linear regression analysis to the adjusted example of Kalotay and Abreo.

	R^2 [%]	$\widehat{\beta}_0$	$\widehat{\beta}_1$	effective
fair value	94.57	99,998,433.08	-2.59	*no*
cumulative	95.01	-1,713.80	-2.65	*no*
period-by-period	4.75	149.93	-0.25	*no*

Table 4: Results of the application of the hedge effectiveness tests based on time series for the hedge data of Figure 1 on page 5. For each time interval 60 data points where created according to the market value development indicated in Figure 1.

	Expected to be effective	Linear Regression fair values R^2 [%]	β_0	β_1	eff.	Linear Regression cumulative changes R^2 [%]	β_0	β_1	eff.
t_1	yes	100.00	100,000	-1.00	yes	100.00	0.0013	-1.00	yes
t_2	yes	100.00	100,000	-1.00	yes	100.00	0.0000	-1.00	yes
t_3	yes	100.00	100,000	-1.00	yes	100.00	-0.0017	-1.00	yes
t_4	no	96.40	101,884	-0.84	*yes*	96.36	1,936.1733	-0.84	*yes*
t_5	no	98.97	99,649	-0.86	*yes*	98.84	-268.8367	-0.86	*yes*
t_6	no	100.00	100,000	1.00	no	100.00	0.0373	1.00	no
t_7	no	100.00	100,000	1.00	no	100.00	0.1245	1.00	no
t_8	no	100.00	100,000	1.00	no	100.00	0.0000	1.00	no

	Linear Regression period-by-period R^2 [%]	β_0	β_1	eff.	Variability-Reduction VR [%]	eff.	Volatility-Reduction VRM [%]	eff.	Adj. Hedge Interval $[\frac{4}{5}, \frac{5}{4}]$ max.	eff.
t_1	100.00	-0.0028	-1.00	yes	100.00	yes	99.99	yes	1.00	yes
t_2	100.00	0.0022	-1.00	yes	100.00	yes	100.00	yes	1.00	yes
t_3	100.00	-0.0005	-1.00	yes	100.00	yes	100.00	yes	1.00	yes
t_4	34.42	2,073.0826	-0.62	no	23.28	no	73.68	no	560.52	no
t_5	84.92	933.9817	-0.91	*yes*	84.32	*yes*	80.17	*yes*	796.93	no
t_6	100.00	-0.0135	1.00	no	-299.98	no	100.00	no	81.00	no
t_7	100.00	0.0017	1.00	no	-299.99	no	-100.00	no	81.00	no
t_8	100.00	0.0186	1.00	no	-299.99	no	-100.00	no	81.00	no

4.2.3 Variability-Reduction Measure

Finnerty and Grand (2002) develop a test based on the assumption, to regard a hedge as effective if and only if

$$RVR = 1 - \frac{\sum_{i=1}^{n}(-\widehat{\beta}_1\, \Delta SG_i + \Delta GG_i)^2}{\sum_{i=1}^{n} \Delta GG_i^2} \geq 80\% \ ,$$

where $\widehat{\beta}_1$ denotes the estimate obtained from the regression described above with opposed dependent and independent variables.

For the retrospective test instead of $\widehat{\beta}_1$ the "actual hedge ratio the hedger implemented" should be used, which implies for a perfect hedge $\widehat{\beta}_1 = -1$.

Then for period-by-period changes the variability-reduction is defined as

$$VR(\Delta GG, \Delta SG) = 1 - \frac{\sum_{i=1}^{n}(\Delta SG_i + \Delta GG_i)^2}{\sum_{i=1}^{n} \Delta GG_i^2} \ .$$

Finnerty and Grand suggest the following test:

Test 8 *A hedge is effective if the variability-reduction is at least* 80%, *i.e.*

$$VR \geq 80\% \ .$$

For evaluation of the criteria we regard the following example: Let a hedge have a constant decrease of 30,000 US $ in the market value of the hedged item and a constant increase of 20,000 US$ for the hedging instrument, i.e. the period-by-period differences are

$$\Delta GG_i = -30,000\,\text{US\$} \quad \text{and} \quad \Delta GG_i = 20,000\,\text{US\$ for all dates } i.$$

Offsetting is measured by this test, but the problem of large numbers is not avoided, as in this example after n periods we obtain a loss in the hedge position of $n \cdot 10,000$ US$, but the variability-reduction has a constant value of $VR =$ 88.89%. The adjusted example of Kalotay and Abreo (2001) illustrates that the problem of small numbers may occur in this test, as we obtain a value of $VR = 0.0042\%$, which is obviously lower than 80%.

The symmetry is not fulfilled, as in the above example we obtain

$$VR(\Delta GG, \Delta SG) = 88.89\,\% \quad \text{and} \quad VR(\Delta SG, \Delta GG) = 75.00\,\% \ ,$$

which implies the hedge to be effective in the first and ineffective in the second case.

Scalability is fulfilled as one can easily see that a percentage will be canceled in the fraction. So again only the first and the last criteria are met.

4.2.4 Volatility Reduction Measure

One other approach to measure effectiveness is based on the idea of risk reduction, as stated by Hull (2003) "hedge effectiveness can be defined as the proportion of the variance that is eliminated by hedging."

According to Coughlan, Kolb and Emery (2003) the relative risk reduction is defined as

$$RRR = 1 - \frac{\text{risk of portfolio}}{\text{risk of underlying}} \ .$$

As possible risk measures they mention value-at-risk and the variance or volatility of changes in fair value. The latter is used by applying standard deviation of changes in fair value.

Kalotay and Abreo (2001) have developed their test based on volatility reduction: "The volatility of the item being hedged in the *absence* of a hedge is the obvious point of reference against which this reduction should be measured." This measurement is detailed in the following test.

Test 9 *A hedge is effective, if the volatility reduction measure*

$$VRM = 1 - \frac{\sigma_{\Delta GP}}{\sigma_{\Delta GG}} = 1 - \frac{\sigma_{\Delta GG + \Delta SG}}{\sigma_{\Delta GG}}$$

is part of the interval [80%, 125%].

Corresponding to the examples described by Coughlan, Kolb and Emery (2003) we use cumulative changes for the differences.

Coughlan, Kolb and Emery (2003) suggest other thresholds. They regard a hedge as effective, if $RRR \geq 40\%$, as they indicate that "a correlation of -80% corresponds to a level of risk reduction of approximately 40%".

For evaluation of Test 9 we regard the following example: Let for date i the cumulative differences in the market value of the hedging instrument be $\Delta SG_i = i \cdot 10{,}000$ US$ and let $\Delta GG_i = -5/4 \cdot \Delta SG_i$ for all dates i.

For any period this results in

$$VRM(\Delta GG, \Delta SG) = 80.00\,\%$$

which implies the hedge to be effective. Obviously this test measures offsetting, but the example illustrates that the maximum loss is not limited. The problem of small numbers may occur as well. Using the expansion of the example of Kalotay and Abreo illustrated in Figure 3, we obtain

$$VRM = 1 - \frac{1,742.14}{2,715.71} = 35,85\% ,$$

which results in an ineffective hedge.

Further on this test is not symmetric, as changing ΔSG and ΔGG in the above example results in a value of $VRM = 75.00\,\%$ for all periods, which implies the hedge to be ineffective. As this test is scalable, it meets the first and last criterion.

When implementing this test further non-mathematical management considerations are necessary, as this method for determining effectiveness is subject of United States Patent Application 20020032624.

5 Adjusted Hedge Interval

In Section 3, we have formulated criteria which should naturally be met by an effectiveness test. As summarized in Table 5, none of the tests we presented fulfills all of these criteria. According to the geometrical interpretation of the criteria it seem obvious that the effective area has to be similar to the dollar offset ratio except for large and for small numbers: For large numbers it should be parallel to the northwest-southeast diagonal and for small numbers it should broaden the intersection of the two cones.

Suppose h_1 and h_2 are natural numbers with $h_1 < h_2$. A test which is in medium scale equivalent to the dollar offset ratio based on the interval $[\frac{h_1}{h_2}, \frac{h_2}{h_1}]$ can then be obtained with the following generalized hedge interval:

Let auxiliary functions $f_1, f_2 : I\!R \to I\!R$ be defined with

$$\begin{aligned} f_1(\Delta GG) &= \frac{h_1^2 + h_2^2}{2h_1 h_2} \Delta GG \quad \text{and} \\ f_2(\Delta GG) &= \frac{h_1^2 - h_2^2}{2h_1 h_2} \sqrt{(\Delta GG)^2 + c}\,. \end{aligned}$$

Table 5: Results of the evaluation of effectiveness test according to the criteria defined in Section 3.1 and Section 3.2. We indicate with the symbol ($\surd$), when a criterion is fulfilled only if particular constants are chosen for the measurement.

(i) Offsetting (ii) Large Numbers (iii) Small Numbers
(iv) Symmetry (v) Scalability (vi) Smooth Transactions

		(i)	(ii)	(iii)	(iv)	(v)	(vi)
Tests based on two points of data							
1	Dollar offset ratio	$\surd$	–	–	$\surd$	$\surd$	$\surd$
2	Intuitive response	$\surd$	–	$\surd$	$\surd$	($\surd$)	–
3	Lipp modulated dollar offset	$\surd$	–	–	$\surd$	($\surd$)	–
4	Schleifer-Lipp modulated offset	$\surd$	–	–	$\surd$	($\surd$)	–
5	Gürtler effectiveness test	–	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
6	Hedge interval	$\surd$	–	$\surd$	$\surd$	($\surd$)	$\surd$
10	Adjusted hedge interval	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
Tests based on time series of data							
7	Linear regression (fair value)	$\surd$	–	–	–	$\surd$	
	Linear reg. (cumulative changes)	$\surd$	–	–	–	$\surd$	
	Linear reg. (period-by-period)	$\surd$	–	–	–	$\surd$	
8	Variability-reduction measure	$\surd$	–	–	–	$\surd$	
9	Volatility reduction measure	$\surd$	–	–	–	$\surd$	
10	Adjusted hedge interval based on 100 % compliance level	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	

We obtained the simple formula for our hedge interval (Test 6) from the geometrical interpretation by asserting

$$\underline{f} = -f_1 + f_2 \quad \text{and} \quad \overline{f} = -f_1 - f_2 \, .$$

This is equivalent to a hedge being effective, if

$$\left| \frac{2h_1h_2 \; \Delta SG + (h_1^2 + h_2^2) \; \Delta GG}{\sqrt{\Delta GG^2 + c}} \right| \leq h_2^2 - h_1^2 \, .$$

For example, for the underlying interval $[\frac{9}{10}, \frac{10}{9}]$ this implies to regard a hedge effective if

$$\left| \frac{180 \; \Delta SG + 181 \; \Delta GG}{\sqrt{\Delta GG^2 + c}} \right| \leq 19 \, .$$

To adjust this interval to limit maximum loss or gain of the hedging position as part of criterion (i), we want to let the bounding functions of the effective area be parallel to the northwest-southeast diagonal. The maximum distance of this line should be a fixed percentage p of $\frac{1}{\sqrt{2}} GP_0$ to ensure scalability of the test. In our examples we used a value of $p = 25\%$. So for "large" values of ΔSG we regard the bounding functions $\underline{g}(\Delta GG) = -\Delta GG - p\,GP_0$ and $\overline{g}(\Delta GG) = -\Delta GG + p\,GP_0$ instead of $\underline{f}$ and $\overline{f}$.

Let auxiliary terms d and e be defined with $d := p\, GP_0$ and

$$e := \sqrt{d^2\, h_2^2\, h_1^4 + 2\, d^2\, h_2^3\, h_1^3 + h_2^4\, d^2\, h_1^2 - h_1^5\, h_2\, c + 2\, h_1^3\, h_2^3\, c - h_1\, h_2^5\, c}\,.$$

Then the function $\underline{f}$ and $\underline{g}$ intersect at $\Delta GG = x_1$ and $\Delta GG = x_2$, where

$$x_1 = \frac{-4\, d\, h_2\, h_1^2 + 4\, h_2^2\, d\, h_1 + 4\, e}{8\, h_1\, h_2\, (h_1 - h_2)}$$

and

$$x_2 = \frac{-4\, d\, h_2\, h_1^2 + 4\, h_2^2\, d\, h_1 - 4\, e}{8\, h_1\, h_2\, (h_1 - h_2)}\,.$$

Analogously the function $\overline{f}$ and $\overline{g}$ intersect at $\Delta GG = -x_2$ and $\Delta GG = -x_1$. To satisfy criterion (vi) we use these points as turning points. Let X_L and X_U be intervals defined with $X_L = [x_1, x_2]$ and $X_U = [-x_2, -x_1]$.

Then, more precisely, we assume a hedge to be effective if and only if

$$\underline{f}(\Delta GG) \leq \Delta SG \leq \overline{f}(\Delta GG)$$

where

$$\underline{f}(\Delta GG) = \begin{cases} -f_1(\Delta GG) + f_2(\Delta GG) & \text{for } \Delta GG \in X_L \\ -\Delta GG - p\, GP_0 & \text{else} \end{cases}$$

and

$$\overline{f}(\Delta GG) = \begin{cases} -f_1(\Delta GG) - f_2(\Delta GG) & \text{for } \Delta GG \in X_U \\ -\Delta GG + p\, GP_0 & \text{else.} \end{cases}$$

This can simply and equivalently be formulated by the following criterion.

Test 10 *(AHI – adjusted Hedge Interval) Let h_1 and h_2 be natural numbers with $h_1 \leq h_2$ representing an underlying dollar offset interval $[\frac{h_1}{h_2}, \frac{h_2}{h_1}]$. Let c be a fixed percentage of GP_0^2, i.e. $c = c_{GP_0^2}$.*

A hedge is effective if and only if $|GP_t - GP_0| \leq p\, GP_0$ and

$$\left| \frac{2h_1 h_2\ \Delta SG + (h_1^2 + h_2^2)\ \Delta GG}{\sqrt{\Delta GG^2 + c}} \right| \leq h_2^2 - h_1^2\,.$$

The effective area is illustrated in Figure 7 with a value of $p = 25\%$ and of $c = 10^{-7} \cdot GP_0^2 = 1000$ for large scale. It is compared with different underlying dollar offset intervals in Figure 8 on page 30.

For the example introduced in Figure 1 we obtain the expected results when regarding just two points of time and when applying a 100 % compliance level for the time series. For these we provide in Table 4 the maximum value of the fraction. For the periods $t_3 - t_4$, $t_4 - t_5$ and $t_7 - t_8$ the additional criterion $|GP_t - GP_0| \leq p\, GP_0$ implies ineffectiveness as well. For the adjusted example of Kalotay and Abreo we obtain a maximum value of 7.5378, which implies the hedge to be effective as expected.

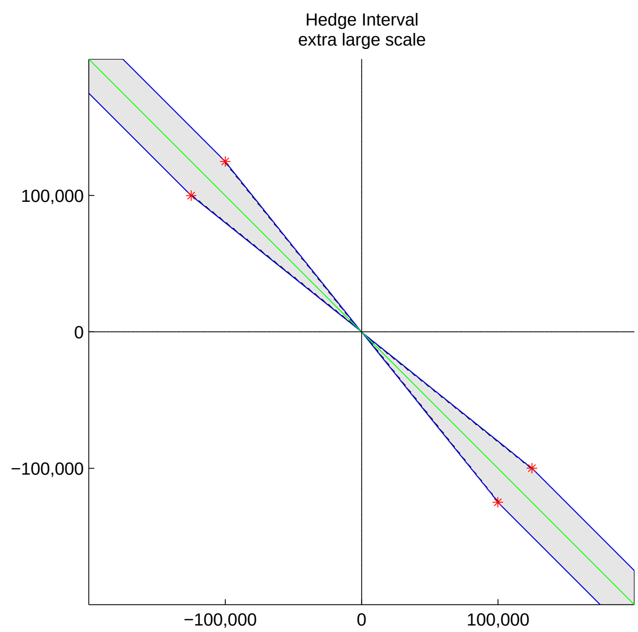


Figure 7: Illustration of the adjusted hedge interval, the points where the bounding functions $\underline{f}$ and $\overline{f}$ change from determining cones to parallels to the northwest-southeast diagonal are marked with an '$*$'.

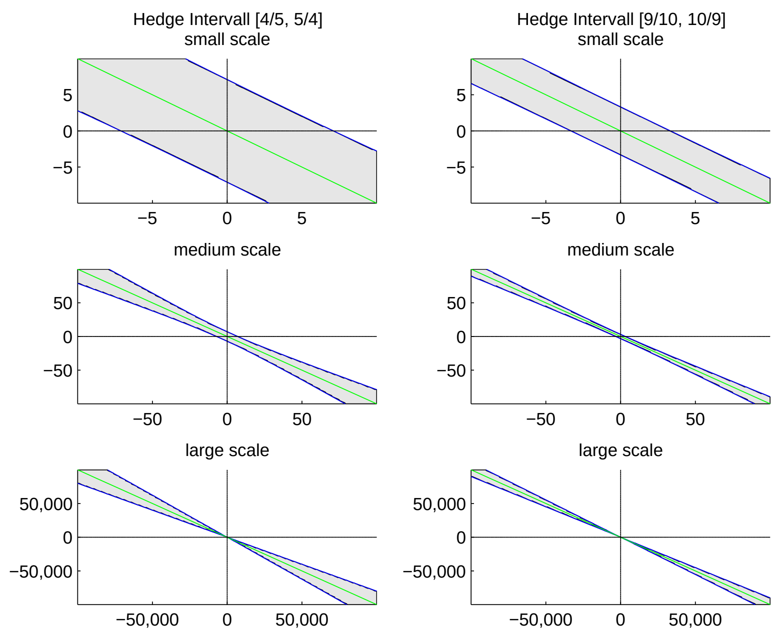


Figure 8: Illustration of the adjusted hedge interval for an underlying dollar offset interval of $[\frac{4}{5}, \frac{5}{4}]$ and $[\frac{9}{10}, \frac{10}{9}]$.

Theorem 1 *The adjusted hedge interval test meets criteria (i) to (vi).*

Proof: Criteria (i) is fulfilled as the adjusted hedge interval approximates the dollar offset ratio in a medium scale, i.e. for most expected market developments.

For *large values* the additional inequality $|GP_t - GP_0| \leq p\,GP_0$ limits the maximum possible gain or loss and for *small values* the area of effectiveness is enlarged.

The *symmetry* is guaranteed, as one can easily show that

$$\underline{f}(x) = \underline{f}^{-1}(x) \quad \text{and} \quad \underline{f}(-x) = -\overline{f}(x) \text{ for all } x \in I\!R\,.$$

For large values of ΔGG the *scalability* is obvious and for small values the factor α can be canceled in the fraction

$$\frac{2h_1h_2\,\alpha\,\Delta SG + (h_1^2 + h_2^2)\,\alpha\,\Delta GG}{\sqrt{(\alpha \Delta GG)^2 + \alpha^2\, c_{GP_0^2}}}\,.$$

The last criterium is a direct consequence of the construction of the functions $\underline{f}$ and $\overline{f}$, guaranteeing that they are *continuous.* □

In summary, according to the criteria defined in Section 3 and to get a test as simple as possible, the adjusted hedge interval approach presents a geometrically natural enhancement of the dollar offset ratio. Both problems of the dollar offset ratio concerning small and large numbers are avoided. Currently no limitations are known.

FAS 133 §230 (Appendix "Background Information and Basis for Conclusions") contains the following statement: "Because hedge accounting is elective and relies on management's intent, it should be limited to transactions that meet reasonable criteria."

While there appears to be no definitive answer to the question of what is or is not reasonable, the measurable criteria proposed in this paper may form at least parts of one.

References

Guy Coughlan, Johannes Kolb, Simon Emery (2003). *HEAT™ Technical Document: A consistent framework for assessing hedge effectiveness under IAS 39 and FAS 133.* Credit & Rates Markets, J. P. Morgan Securities Ltd., London.

Financial Accounting Standards Board (1998). *Statement of Financial Accounting Standards No. 133, Accounting for Derivative Instruments and Hedging Activities.* Norwalk, Connecticut.

Financial Accounting Standards Board (2000). Derivatives Implementation Group. *Statement 133 Implementation Issue No. E7 and E8.*

John D. Finnerty, Dwight Grand (2002). *Alternative Approaches to Testing Hedge Effectiveness.* Accounting Horizons, Vol. 16, No. 2, pp. 95-108.

Marc Gürtler (2004). *IAS 39: Verbesserte Messung der Hedge-Effektivität.* Zeitschrift für das gesamte Kreditwesen, 57(11), pp. 586-588.

Angelika C. Hailer, Siegfried M. Rump (2003). *Hedge Effektivität: Lösung des Problems der kleinen Zahlen.* Zeitschrift für das gesamte Kreditwesen, 56(11), pp. 599-603.

John C. Hull (2003). *Options, Futures and other Derivatives.* Fifth ed, Prentice Hall, ISBN 0-13-046592-5.

IASCF (2003). *International Accounting Standard 39, Financial Instruments: Recognition and Measurement.* ISBN 1-904230-35-0.

IASCF (2003). *Guidance on Implementing IAS 39, Financial Instruments: Recognition and Measurement.* ISBN 1-904230-36-9.

Andrew Kalotay, Leslie Abreo (2001). *Testing Hedge Effectiveness for FAS 133: The Volatility Reduction Measure.* Journal of Applied Corporate Finance, 13, pp. 93-99.

Ira G. Kawaller, Paul D. Koch (2000). *Meeting the "Highly Effective Expectation" Criterion for Hedge Accounting.* Journal of Derivatives 7, pp 79 - 87.

Ira G. Kawaller (2002). *Hedge Effectiveness Testing. Using Regression Analysis.* Association for Financial Professionals, AFP Exchange.

Louis Schleifer (2001). *A New Twist To Dollar Offset.* FAS133.com, International Treasurer.

	hedged item	hedging instrument		hedged item	hedging instrument
t_0	100000000.000000	0.000000	t_{30}	100003357.574985	−2193.890429
t_1	100000103.001900	−103.001900	t_{31}	100003128.434061	−1948.033513
t_2	100000353.590078	−353.590078	t_{32}	100003534.893647	−2116.549673
t_3	100000494.168988	−494.168988	t_{33}	100003628.644687	−1970.821094
t_4	100000598.016519	−598.016519	t_{34}	100003622.368034	−2207.252223
t_5	100000657.772338	−657.772338	t_{35}	100003917.423414	−2277.464310
t_6	100000687.731707	−687.731707	t_{36}	100003993.381991	−2158.908143
t_7	100000799.662261	−799.662261	t_{37}	100004310.907953	−2341.034814
t_8	100000842.792666	−842.792666	t_{38}	100004103.328372	−2524.991129
t_9	100001028.698458	−1028.698458	t_{39}	100004433.719249	−2393.433839
t_{10}	100001064.007115	−1064.007115	t_{40}	100004672.190331	−2384.594722
t_{11}	100001389.824539	−1172.595493	t_{41}	100004816.973781	−2770.884811
t_{12}	100001557.664208	−1150.782722	t_{42}	100005078.291744	−2566.504957
t_{13}	100001698.732443	−1255.067048	t_{43}	100005234.785690	−2748.907485
t_{14}	100001630.232339	−1511.406422	t_{44}	100005521.315318	−2631.107353
t_{15}	100001759.650949	−1451.931077	t_{45}	100005591.490503	−2827.301971
t_{16}	100001869.897716	−1273.081358	t_{46}	100005739.268130	−2810.749918
t_{17}	100001842.315042	−1314.335776	t_{47}	100006060.092710	−2990.638158
t_{18}	100001946.738263	−1551.608484	t_{48}	100006253.362845	−3109.586186
t_{19}	100001955.393141	−1393.684949	t_{49}	100006672.790732	−3226.428025
t_{20}	100001971.697311	−1764.244530	t_{50}	100006860.993342	−3297.429641
t_{21}	100002229.684999	−1683.357540	t_{51}	100006966.126384	−3176.120296
t_{22}	100002174.248184	−1537.209193	t_{52}	100007284.084244	−3403.845824
t_{23}	100002548.133024	−1897.389802	t_{53}	100007659.784692	−3385.999102
t_{24}	100002319.987673	−1885.544774	t_{54}	100007916.150862	−3621.853063
t_{25}	100002701.578312	−1900.691349	t_{55}	100008136.357592	−3484.435480
t_{26}	100002811.340841	−1943.956280	t_{56}	100008753.285949	−3695.540754
t_{27}	100002668.086171	−1824.548991	t_{57}	100008871.361747	−3588.127324
t_{28}	100002833.722169	−2000.808038	t_{58}	100009441.079760	−3677.904363
t_{29}	100003188.612390	−2074.794230	t_{59}	100009657.356320	−3871.214709
			t_{60}	100010000.000000	−4000.000000

Market values used for the adjusted example of Kalotay and Abreo.

$t_0 - t_1$	$t_1 - t_2$	$t_2 - t_3$	$t_3 - t_4$	$t_4 - t_5$	$t_5 - t_6$	$t_6 - t_7$	$t_7 - t_8$
100000.000000	100999.900000	96000.000000	99999.900000	500000.000000	100000.000000	95833.000000	91666.000000
100285.808944	100515.386757	96262.796238	104924.505902	459039.354647	99795.421604	96045.936419	92180.988738
99927.052332	100508.782704	95761.222659	78411.605183	468176.341463	100099.797773	96003.844912	91387.053237
100064.859904	100600.757412	96688.375247	60715.739185	500000.000000	99821.030597	95308.698692	91939.919875
99676.490864	100342.949388	96003.229271	122689.652977	400268.179049	99584.631300	95133.323625	91309.825654
99822.640455	100415.952105	95868.129637	124522.970444	360978.332222	100124.809732	94780.272926	91807.065507
100145.217081	100649.464975	95916.686899	63439.552226	386523.407302	99119.741635	95428.381665	90823.231450
100600.128560	100616.965630	96185.702715	126764.862937	360106.263099	98872.601815	94757.282682	91306.206801
99814.172497	100480.753888	96603.700788	69512.725588	322112.610223	99307.573589	95319.914382	90608.710429
100207.823754	100203.124372	96611.842934	59160.145655	303952.458581	100031.260881	94755.240714	91626.329508
100054.042228	99925.346105	96963.985821	126810.468189	267819.444486	98864.711031	94651.321577	90784.709143
99894.484996	100258.425094	96602.251780	126068.643553	267289.907815	99077.684569	95509.121278	90223.144755
99781.556521	99644.475142	96993.503406	145024.451593	219728.299859	98879.921532	95120.598632	91077.806191
99982.109703	99830.154658	96421.235298	142161.492523	227381.241884	99663.104087	94873.174159	90167.924848
100560.998262	99943.069494	96722.497047	85918.359013	236672.712772	99127.501571	94527.028673	91200.238431
100348.505651	99525.999070	97104.970521	130911.242960	164146.985801	99126.706645	94665.704506	91181.185389
99835.689623	99597.423763	96840.152861	65612.950113	114887.014546	99350.065851	94747.608788	90874.773669
99871.202340	99614.370643	97384.566309	132609.875154	111441.488229	98346.437119	94699.475153	89826.623108
100394.784030	99279.939539	97280.384165	132920.618626	100695.866079	98268.921173	95130.230177	90974.749437
99928.352164	99268.452581	96854.723282	50570.350949	100000.000000	98740.924717	94562.363722	90623.391360
100781.057848	99010.197564	97172.565981	106020.814305	100000.000000	97964.110021	94600.588037	89524.119343
100503.028478	98763.431128	97711.133356	115995.105114	83514.443286	99027.505022	94620.028073	90556.848137
100059.677570	99112.940159	97152.249895	118578.168946	113462.610831	97945.781049	93781.160440	90086.025165
100879.930225	98670.644233	97430.498451	113492.061307	135995.554347	98016.643788	93840.639471	90307.282867
100767.848101	99293.086232	97993.305218	110675.944879	93322.719970	98402.196084	93504.415214	89702.277903
100720.921222	98401.551408	97987.740152	90847.379625	67699.168571	98085.321775	93983.131897	89233.114108
100741.912156	99025.106979	97655.468852	138025.238305	82605.136790	97551.992549	94674.487186	89431.297126
100353.788877	98959.480823	97700.574863	146187.020323	50055.269505	97918.511622	93789.933087	89502.063639
100702.340889	98251.324202	98027.780801	104140.104534	53836.359285	98398.434057	93412.060611	90181.481718
100792.662003	98305.016045	97890.904763	99999.900000	144113.783454	98181.077543	93116.986904	90161.393844
100993.149802	98821.059224	98082.422631	99999.900000	113968.882908	98342.023260	94104.412488	90204.329592
100278.585510	98861.077211	98557.747048	137348.090282	95148.718627	97848.293429	93360.094262	89751.836579
100597.222212	97894.191864	98642.252165	161556.079873	123823.043013	97892.379068	93544.533713	88942.051162
100127.451523	98504.207696	98424.547155	104093.694988	90265.515207	97746.594909	92899.530713	88635.965544
100610.219218	98319.878683	98792.164544	181384.296813	145072.200340	97470.377354	93137.057969	89921.502286
100674.511549	97872.415051	98488.854745	154382.309485	92894.250562	98156.230070	93746.667324	88728.827548
100263.887057	98164.704970	98257.500981	136069.799288	79914.276676	97746.041115	93366.590439	88936.551063
100601.903436	97895.145206	98915.569403	180058.527083	53771.174128	97264.464000	93685.177410	88952.001992
100941.356032	98214.844301	99040.868456	173306.652582	74948.176806	97801.340460	92500.076380	89324.836299
100488.813379	97206.546418	98531.606214	273145.131593	76000.695462	97107.097906	93288.589633	88445.837421
100339.779860	97527.345607	98603.256249	274229.979795	106812.661439	97562.360003	93648.094217	89158.318943
101175.206249	97601.896963	99263.831708	219848.493211	86573.404368	97014.570007	92589.453374	88401.894892
100530.516059	97425.137400	99331.031507	272500.577854	69697.365977	97058.564271	92324.357103	88463.648120
100849.259660	97521.953605	99225.106767	240278.117278	141611.899300	97046.516211	93106.653059	89017.630438
100413.892118	97338.254421	98896.057799	269612.575375	55453.721209	96869.387799	93085.749403	88404.522362
100759.822709	97460.467081	99493.190097	288164.144288	58405.754011	96238.284702	92061.767097	88330.741677
100570.111658	97021.866008	99250.998063	336078.413571	89801.257636	97009.246771	93114.389906	88184.519062
100867.638143	96756.563804	99054.909852	367879.243364	124643.942127	96855.351015	91910.785602	88781.279741
101008.860599	96902.361629	99036.521511	382714.566506	62654.919446	96917.281583	92175.506116	88076.990757
101140.771676	96706.914179	99129.281453	392957.299892	98659.929000	96702.017147	91901.521101	88659.891268
101153.941725	97053.627206	99805.666702	383616.885342	76734.935087	96808.064963	92725.455457	88073.050072
100682.097115	96798.134172	99503.185528	354057.574100	107777.341388	96586.768830	92311.474985	87885.907362
101078.503396	96309.879825	99583.507296	404423.253023	53863.000114	96364.494804	92765.267734	87728.580853
100856.541400	96327.618655	99739.880053	385647.881724	142794.076145	95988.110897	91617.367186	87670.710080
101264.289736	96462.625212	100007.319061	434957.482376	94659.203197	96500.358316	91952.237998	87321.599977
100734.211688	96271.147559	99553.075689	449092.134389	120801.607886	96702.188680	92222.258928	88461.282909
101070.684627	95960.021552	99597.301926	463551.791147	64203.648692	96168.779531	92074.582375	87974.934090
100605.993480	96610.305542	99636.801040	500000.000000	116424.486131	96066.085500	91314.694961	87248.953858
100629.297412	95981.874982	99953.720704	446045.910627	81979.614963	96629.184245	91938.729800	87839.617883
100999.900000	96000.000000	99999.900000	500000.000000	144655.557868	95833.000000	91666.000000	87500.000000

Market values used for the **hedged item** of the example illustrated in Figure 1.

$t_0 - t_1$	$t_1 - t_2$	$t_2 - t_3$	$t_3 - t_4$	$t_4 - t_5$	$t_5 - t_6$	$t_6 - t_7$	$t_7 - t_8$
0.000000	-1000.070000	4000.000000	0.070000	-450000.000000	0.000000	-4166.000000	-8333.000000
-285.808944	-515.386757	3737.203762	-4924.505902	-426146.249077	-204.578396	-3954.063581	-7819.011262
72.947668	-508.782704	4238.777341	21588.394817	-396075.632769	99.797773	-3996.155088	-8612.946763
-64.859904	-600.757412	3311.624753	39284.260815	-450000.000000	-178.969403	-4691.301308	-8060.080125
323.509136	-342.949388	3996.770729	-22689.652977	-366925.270911	-415.368700	-4866.676375	-8690.174346
177.359545	-415.952105	4131.870363	-24522.970444	-292912.945458	124.809732	-5219.727074	-8192.934493
-145.217081	-649.464975	4083.313101	36560.447774	-347719.910903	-880.258365	-4571.618335	-9176.768550
-600.128560	-616.965630	3814.297285	-26764.862937	-309665.663840	-1127.398185	-5242.717318	-8693.793199
185.827503	-480.753888	3396.299212	30487.274412	-289638.635361	-692.426411	-4680.085618	-9391.289571
-207.823754	-203.124372	3388.157066	40839.854345	-268337.312056	31.260881	-5244.759286	-8373.670492
-54.042228	74.653895	3036.014179	-26810.468189	-230158.281226	-1135.288969	-5348.678423	-9215.290857
105.515004	-258.425094	3397.748220	-26068.643553	-159561.325440	-922.315431	-4490.878722	-9776.855245
218.443479	355.524858	3006.496594	-45024.451593	-143098.289839	-1120.078468	-4879.401368	-8922.193809
17.890297	169.845342	3578.764702	-42161.492523	-154728.727023	-336.895913	-5126.825841	-9832.075152
-560.998262	56.930506	3277.502953	14081.640987	-122106.222113	-872.498429	-5472.971327	-8799.761569
-348.505651	474.000930	2895.029479	-30911.242960	-50687.247197	-873.293355	-5334.295494	-8818.814611
164.310377	402.576237	3159.847139	34387.049887	-29948.807526	-649.934149	-5252.391212	-9125.226331
128.797660	385.629357	2615.433691	-32609.875154	-24499.238491	-1653.562881	-5300.524847	-10173.376892
-394.784030	720.060461	2719.615835	-32920.618626	-14591.442103	-1731.078827	-4869.769823	-9025.250563
71.647836	731.547419	3145.276718	49429.649051	0.000000	-1259.075283	-5437.636278	-9376.608640
-781.057848	989.802436	2827.434019	-6020.814305	0.000000	-2035.889979	-5399.411963	-10475.880657
-503.028478	1236.568872	2288.866644	-15995.105114	16485.556714	-972.494978	-5379.971927	-9443.151863
-59.677570	887.059841	2847.750105	-18578.168946	-13462.610831	-2054.218951	-6218.839560	-9913.974835
-879.930225	1329.355767	2569.501549	-13492.061307	-35995.554347	-1983.356212	-6159.360529	-9692.717133
-767.848101	706.913768	2006.694782	-10675.944879	6677.280030	-1597.803916	-6495.584786	-10297.722097
-720.921222	1598.448592	2012.259848	9152.620375	32300.831429	-1914.678225	-6016.868103	-10766.885892
-741.912156	974.893021	2344.531148	-38025.238305	17394.863210	-2448.007451	-5325.512814	-10568.702874
-353.788877	1040.519177	2299.425137	-46187.020323	49944.730495	-2081.488378	-6210.066913	-10497.936361
-702.340889	1748.675798	1972.219199	-4140.104534	46163.640715	-1601.565943	-6587.939389	-9818.518282
-792.662003	1694.983955	2109.095237	0.100000	-44113.783454	-1818.922457	-6883.013096	-9838.606156
-993.149802	1178.940776	1917.577369	0.070000	-13968.882908	-1657.976740	-5895.587512	-9795.670408
-278.585510	1138.922789	1442.252952	29566.577297	4851.281373	-2151.706571	-6639.905738	-10248.163421
-597.222212	2105.808136	1357.747835	-50352.462912	-23823.043013	-2107.620932	-6455.466287	-11057.948838
-127.451523	1495.792304	1575.452845	-61562.302371	9734.484793	-2253.405091	-7100.469287	-11364.034456
-610.219218	1680.121317	1207.835456	-110367.683377	-45072.200340	-2529.622646	-6862.942031	-10078.497714
-674.511549	2127.584949	1511.145255	-82853.473310	7105.749438	-1843.769930	-6253.332676	-11271.172452
-263.887057	1835.295030	1742.499019	-83110.744343	20085.723324	-2253.958885	-6633.409561	-11063.448937
-601.903436	2104.854794	1084.430597	-78499.488530	46228.825872	-2735.536000	-6314.822590	-11047.998008
-941.356032	1785.155699	959.131544	-136657.982632	25051.823194	-2198.659540	-7499.923620	-10675.163701
-488.813379	2793.453582	1468.393786	-162991.404327	23999.304538	-2892.902094	-6711.410367	-11554.162579
-339.779860	2472.654393	1396.743751	-142760.916099	-6812.661439	-2437.639997	-6351.905783	-10841.681057
-1175.206249	2398.103037	736.168292	-121677.258014	13426.595632	-2985.429993	-7410.546626	-11598.105108
-530.516059	2574.862600	668.968493	-178192.636148	30302.634023	-2941.435729	-7675.642897	-11536.351880
-849.259660	2478.046395	774.893233	-227091.061892	-41611.899300	-2953.483789	-6893.346941	-10982.369562
-413.892118	2661.745579	1103.942201	-246628.520577	44546.278791	-3130.612201	-6914.250597	-11595.477638
-759.822709	2539.532919	506.809903	-274754.307566	41594.245989	-3761.715298	-7938.232903	-11669.258323
-570.111658	2978.133992	749.001937	-282747.980880	10198.742364	-2990.753229	-6885.610094	-11815.480938
-867.638143	3243.436196	945.090148	-250568.362730	-24643.942127	-3144.648985	-8089.214398	-11218.720259
-1008.860599	3097.638371	963.478489	-291390.451458	37345.080554	-3082.718417	-7824.493884	-11923.009243
-1140.771676	3293.085821	870.718547	-317955.288516	1340.071000	-3297.982853	-8098.478899	-11340.108732
-1153.941725	2946.372794	194.333298	-279734.123861	23265.064913	-3191.935037	-7274.544543	-11926.949928
-682.097115	3201.865828	496.814472	-366343.283072	-7777.341388	-3413.231170	-7688.525015	-12114.092638
-1078.503396	3690.120175	416.492704	-348299.868415	46136.999886	-3635.505196	-7234.732266	-12271.419147
-856.541400	3672.381345	260.119947	-370075.530164	-42794.076145	-4011.889103	-8382.632814	-12329.289920
-1264.289736	3537.374788	-7.319061	-345855.076575	5340.796803	-3499.641684	-8047.762002	-12678.400023
-734.211688	3728.852441	446.924311	-392809.237381	-20801.607886	-3297.811320	-7777.741072	-11538.717091
-1070.684627	4039.978448	402.698074	-446606.603879	35796.351308	-3831.220469	-7925.417625	-12025.065910
-605.993480	3389.694458	363.198960	-450000.000000	-16424.486131	-3933.914500	-8685.305039	-12751.046142
-629.297412	4018.125018	46.279296	-450035.861708	18020.385037	-3370.815755	-8061.270200	-12160.382117
-1000.070000	4000.000000	0.070000	-450000.000000	0.000000	-4166.000000	-8333.000000	-12500.000000

Market values used for the **hedging instrument** of the example illustrated in Figure 1.