100 € Problem

Notation: For $A \in \mathbb{R}^{n\times n}$, $x \in \mathbb{R}^n$ denote by $|A|, |x|$ the matrix, vector of absolute values.
Hence $|A| \in \mathbb{R}^{n\times n}_{\geq 0}, |x| \in \mathbb{R}^n_{\geq 0}$.

Comparison is entrywise, i.e., for $x, y \in \mathbb{R}^n$,
$x \geq y \;\; :\Leftrightarrow \;\; x_i \geq y_i$ for $i \in \{1, \ldots, n\}$.

Denote $e \in \mathbb{R}^n$ with $e := (1, \ldots, 1)^T$.

Conjecture: Let $A \in \mathbb{R}^{n\times n}$ with $|A|e = ne \;\; \Rightarrow$
$\exists\, 0 \neq x \in \mathbb{R}^n : |Ax| \geq |x|$.

For the first proof or counterexample I am happy to reward you with 100 €.

Note: Theorem 5.8 in [Ru97] proved
$A \in \mathbb{R}^{n\times n}$ with $|A|e = ne \;\; \Rightarrow \;\; \exists\, 0 \neq x \in \mathbb{R}^n : |Ax| \geq \frac{1}{3+2\sqrt{2}}|x|$.